The Mathematics of Directional Drilling

Curriculum for Excellence:
MTH 3-17b Bearings; MTH 4-16a Trig; MTH 4-16b & MTH 4-17a Circles; MTH 4-15a Modelling

National 4 and National 5 Course Guidance & Aims:
“Using mathematics enables us to model real-life situations and make connections and informed predictions. It equips us with the skills we need to interpret and analyse information, simplify and solve problems, assess risk and make informed decisions.”

This exercise uses Pythagoras, Trigonometry and the Geometry of a Circle to plan a directional well to thousands of metres below the sea-bed.

**FIRST BIT: LOCATIONS**
The relative locations of the Target and Relief Wells are calculated.

**SECOND BIT: BUILD-UP**
A vertical section is drilled, then, starting at the Kick-Off Point, the angle of incidence to the vertical is gradually increased to a maximum. This is reached at End Of Build.

**FINAL BIT: TANGENT**
The direction is maintained along the Tangent Section to intersect at a point above the Target reservoir

**EXTRA BIT:**
Total length of borehole is calculated.

- Reservoir depth = 7000m
INTRODUCTION

There are many reasons why it may not be possible or desirable to drill a vertical well. On land there may be cities, towns or lakes to avoid. Offshore and beneath the sea-bed there may be difficult rock formations, geological faults or existing pipeline networks to avoid.

When drilling a well all precautions are taken to ensure that hydrocarbons that may be within pore spaces in rocks deep underground are always under control. This is done by always ensuring that the pressure of the hydrocarbons in the pore spaces is always less than the pressure exerted by the fluid in the wellbore. ‘Drilling mud’ is used to ensure the wellbore is under control. This is a weighted fluid which, as well as keeping the well under control, also lubricates and cools the bit whilst drilling and carries cuttings of rock back to surface.

Planning a directional well has many economic and safety benefits.

Economically directional drilling increases the area of the reservoir accessed, increases hydrocarbon recovery, enables more wells to be drilled from one location and reduces costly rig moves.

Safety-wise, a directional relief well is planned to relieve pressure in a troubled well or to manage a hydrocarbon release as in the recent Elgin incident.

In this exercise you will be planning a relief well from one platform to target the reservoir of a neighbouring platform. Your target is thousands of metres below the sea-bed. Your tools are Pythagoras, trigonometry and the geometry of the circle.

In developing this exercise I would like to thank:

- Polly Tandy, Drilling Engineer with Chevron Upstream Europe for introducing me to the mathematical techniques used in directional drilling. Polly outlined the exercise and provided realistic data.
- Helen Martin, University of Aberdeen, School of Education for help along the way.

I’m sure this exercise will develop further but in the meantime please contact me if you have any questions or suggestions.

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FIRST BIT

THE POSITIONS OF THE TARGET AND RELIEF WELLS

The FIRST BIT consists of 2 questions relating to the relative positioning of the 2 wells.

Data Given:

The Target well is at a depth of 7,000 metres.

The positions of the Target and Relief wells are given using UTM co-ordinates.

<table>
<thead>
<tr>
<th>UTM Co-ordinates</th>
<th>Eastings</th>
<th>Northings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Well</td>
<td>585 407mE</td>
<td>646 4375mN</td>
</tr>
<tr>
<td>Relief Well</td>
<td>580 201mE</td>
<td>646 2751mN</td>
</tr>
</tbody>
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Questions:

1. What is the bearing of the Relief well from the Target well ‘T’?

2. What is the distance D between Relief well and the Target well?
1. What is the bearing of the Relief well from the Target well ‘T’?

\[ \tan^{-1}\left(\frac{5206}{1624}\right) = 3.20566 \]

\[ a = 73^\circ \]

Bearing = 180 + 73 = 253°
2. What is the distance $D$ between Relief well and the Target well?

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$dE = 5206m$  
$dN = 1624m$

Pythagoras

$D^2 = dN^2 + dE^2$

$D^2 = 2637376 + 27102436$

$D^2 = 29739812$

$D = 5453m$

Distance between Relief well and Target Well = 5453m
The SECOND BIT contains questions 3, 4 and 5.

They are concerned with two sections of the well planning: the vertical section and the build-up section.

Data:

Well Planning – Vertical and Build-Up Sections

a. Drill a vertical section to a depth of 200m depth to reach the Kick-Off Point (KOP) and commence the Build-Up section.

b. During the Build-Up section increase the angle of inclination to the vertical by 3° every 10m along the borehole until you reach 40° - End Of Build (EOB).

Questions:

3. What is the True Vertical Depth (TVD) we have reached at End Of Build (EOB)?

4. What is the Measured Depth (MD) along the borehole from the surface to the EOB?

5. What is the Horizontal Displacement (H) from the Relief Well reached at EOB?
3. What is the True Vertical Depth (TVD) we have reached at End Of Build (EOB)?

At EOB:

\[ \text{TVD} = \text{Distance to KOP} + \text{Increase in depth through build up} \]

\[ \text{TVD} = 200 + B \text{ metres} \]

\[ B = R \,(\sin b ^ {\circ}) \]

\[ b = 40 ^ {\circ} \,(\text{alternate angles}) \]

Finding \( R \), the radius of curvature of the arc:

*Relating \( R \) to the build-up rate: \( 3 ^ {\circ} / 10 \text{m along the borehole} \)

\[ R = \frac{360}{2\pi} \times \frac{\text{change in arc length}}{\text{change in angle of incidence}} \text{ metres} \]

\[ R = 57.3 \times \frac{10}{3} \text{ metres} \]

Radius of curvature = 191m

\[ \text{TVD} = 200 + R \sin 40 ^ {\circ} \]

\[ \text{TVD} = 200 + 122.77 = 322.8 \text{m}. \]

At End of Build Section: \( \text{TVD} = 322.8 \text{m} \)
4. What is the Measured Depth (MD) along the borehole from the surface to the EOB?

\[ \text{MD} = \text{TVD} + \text{arc length} \]

\[ \text{MD} = 200 + L \]

**Finding L:**

By proportion: angle to arc length

\[ \frac{3}{10} = \frac{b}{L} \]

\[ L = \frac{10 \times 40}{3} = 133.33\text{m} \]

\[ \text{MD} = 200 + 133.33 \]

**Measured Depth along borehole = 333.33m**
5. What is the Horizontal Displacement (H) from the Relief Well reached at EOB?

\[ H = \text{Horizontal displacement at EOB} \]
\[ R = \text{Radius of curvature (Question 3)} \]
\[ b = 40^\circ \]
\[ \cos b = \frac{(R-H)}{R} \]
\[ R \cos b = R - H \]
\[ H = R (1 - \cos 40) \]
\[ H = 191 \times 0.234 \text{ m} \]

Horizontal Displacement at End Of Build = 44.69 metres
During the FINAL BIT drilling continues, along the tangent from EOB, to intersect the Target Well which is at a depth of 7,000m. Question 6 is critical; the intersection should be above the Target Well.

Data:

Well Planning continued:

- At EOB maintain this direction until you intercept the Target well - above 7,000m

Question:

6. At what depth would the Relief well intersect the Target Well?
6. At what depth would the Relief well intersect the Target Well?

I = Intersection depth
D = distance between the 2 wells (Question 1)

\[ I = TVD + P \]
\[ P = A \tan p \]
\[ I = (200 + A \tan p) \]

\[ A = D - H \]
\[ A = 5453 - 44.69 \]
\[ A = 5408.31m \]

\[ \tan p = \frac{P}{A} \]

Angle \( p = 50^\circ \) (complementary angle)
THE MATHEMATICS OF DIRECTIONAL DRILLING – BIT BY BIT

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\[ P = A \tan 50 \]

\[ P = 5408.31 \times 1.192 \]

\[ P = 6446.70 \text{m} \]

\[ I = \text{TVD} + P \]

\[ I = 322.8 + 6446.7 \]

\[ \text{Intersection Depth} = 6769.5 \text{metres} \]
EXTRA BIT

LENGTH OF TANGENT SECTION and BOREHOLE

The EXTRA BIT rounds off the exercise, working out the length of the Tangent Section before calculating the total length of the borehole.

Question 7:

a. What is the length, along the borehole, of the Tangent Section?

b. Calculate the total length of the Relief Well borehole.
7. What is the length of:
   a. The Tangent section of the borehole, from EOB to intersection?
   b. The total borehole?

\[ \sin p = \frac{P}{T} \]

\[ P = 6446.70 \]

\[ p = 50^\circ \text{ (complementary angle)} \]

\[ T = \frac{6446.70}{\sin 50} \]

Tangent Section length = 8415.62 metres

Length of borehole = 200 + L + T

Length of borehole = 8748.95 metres
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